Lecture - 3: Planetary Motion

Workshop on Mechanics and Python

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References

This lecture is based on

Chapter 9 of The Feynman Lectures in Physics, Volume 1

A Three Dimensional Coordinate System

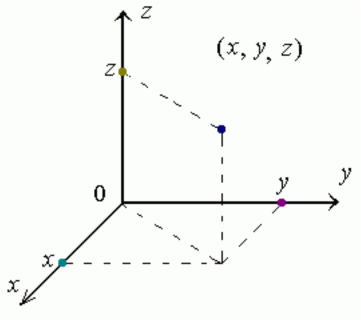


Figure: Three Dimensional Cartesian Coordinate System

Motion in Three Dimensions

Motion of a particle in a cartesian coordinate system

We have to make a table of three entries, giving the position of the particle in three dimensions

$$x = x(t)$$
$$y = y(t)$$
$$z = z(t)$$

t	x	y	z
0	1.0	0.0	0.0
0.1	2.0	0.1	0.5
0.2	2.9	0.7	1.4

Figure: An example of a motion in three dimension

Motion in Three Dimensions: Graph

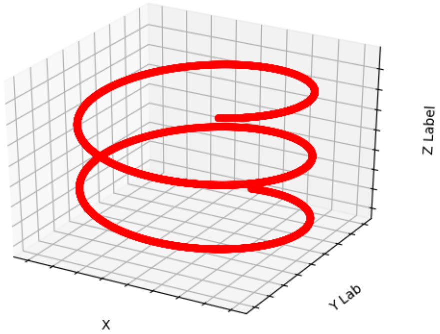


Figure: Motion of a particle in three dimensions

Velocity in Three Dimensions:

Velocity in three dimensional cartesian coordinate system

$$v_x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \equiv \frac{dx}{dt}$$
$$v_y(t) = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} \equiv \frac{dy}{dt}$$
$$v_z(t) = \lim_{\Delta t \to 0} \frac{z(t+\Delta t) - z(t)}{\Delta t} \equiv \frac{dz}{dt}$$

Acceleration in Three Dimensions

Acceleration in three dimensional cartesian coordinate system

$$a_x(t) = \lim_{\Delta t \to 0} \frac{v_x(t+\Delta t) - v_x(t)}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

$$a_y(t) = \lim_{\Delta t \to 0} \frac{v_y(t+\Delta t) - v_y(t)}{\Delta t} \equiv \frac{dv_y}{dt} = \frac{d^2 y}{dt^2}$$

$$a_z(t) = \lim_{\Delta t \to 0} \frac{v_z(t+\Delta t) - v_z(t)}{\Delta t} \equiv \frac{dv_z}{dt} = \frac{d^2 z}{dt^2}$$

Newton's Laws in Three Dimensions

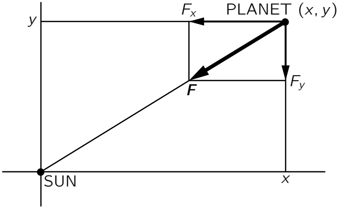
Newton's laws of motion for a particle in three dimensional cartesian coordinate system

$$F_x = \frac{d}{dt}(mv_x) = ma_x$$

$$F_y = \frac{d}{dt}(mv_y) = ma_y$$

$$F_z = \frac{d}{dt}(mv_z) = ma_z$$

Gravitational Force



Force on a planet due to the Sun

The motion is confined to a plane

$$ma_x = -GM_g m_g \frac{x}{r^3}$$
$$ma_y = -GM_g m_g \frac{y}{r^3}$$
$$r = \sqrt{x^2 + y^2}$$

Where m is the inertial mass of the particle while m_g and M_g are the gravitational mass of the particle and the Sun.

Principle of Equivalence

A fundamental property of the gravitational force

The gravitational mass of the particle is same as its inertial mass

$$m_g = m$$

Motion of Earth around the Sun

The one body problem

We will consider the case when $M >> m$ and we can neglect the motion of the Sun. Then in the rest frame of the Sun the motion of Earth is governed by the following equations

$$a_x = -GM\frac{x}{r}$$
$$a_y = -GM\frac{y}{r^3}$$
$$r = \sqrt{x^2 + y^2},$$

together with four initial conditions

$$x(0) = x_0; \quad v_x(0) = v_{x0}$$
$$y(0) = y_0; \quad v_y(0) = v_{y0}.$$