Lecture - 2: Force

Workshop on Mechanics and Python

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References

This lecture is based on

Chapter 8 and Chapter 9 of The Feynman Lectures in Physics, Volume 1

How to describe the change in velocity at a given time t

$t$sec	$v(t)\ \frac{\text{m}}{\text{sec}}$
0	0
1	9.8
2	19.6
3	29.4
4	39.2

Acceleration

Rate of change of velocity

$$a(t) = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t} \equiv \frac{dv}{dt}$$

Acceleration is the second derivative of the position

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) \equiv \frac{d^2x}{dt^2}$$

Rate of change of acceleration

- We can define the rate of change of acceleration in a similar manner
$$\frac{da}{dt} = \lim_{\Delta t \to 0} \frac{a\left(t + \Delta t\right) - a\left(t\right)}{\Delta t}$$
- It is an interesting fact about our description of nature that rate of change of acceleration is not required in formulating the laws of motion

Reference Frame

Laboratory as a reference frame

We will start with a rough idea that we are observing a motion in a laboratory. In this laboratory we have means of measuring position and time of a particle. We have clocks and measuring rods. We will call such a laboratory a reference frame.

One-dimensional coordinate system

A coordinate system is a way of labeling the points of the space

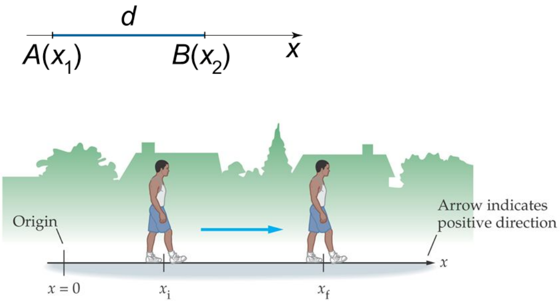

Figure: An example of one dimensional coordinate system

Newton's laws of motion in one-dimension

Newton's second law of motion

The motion of a particle in one dimension when observed from special reference frames satisfies the equation

$$F_x = \frac{d}{dt}(mv_x) = \frac{dp_x}{dt} = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2},$$

where m is the mass of the particle and is a measure of the resistance of the particle to change its momentum. F_x is the force acting on the particle.

Force

Forces are given by the laws of nature

- $F_{\text{spring}}(x) = -k \times x$, where k is a constant we determine from experiments
- $F_{\text{friction}}(v_x) = -\gamma \frac{dx}{dt}$, where γ is a constant we determine from experiments
- These are examples of approximate laws, they are consequence of fundmental forces acting on many many particles.

Mass on the spring

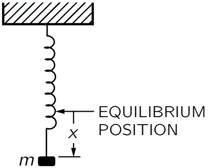

Figure: Mass on the spring

Equation of motion

$$-x = \frac{d^2x}{dt^2}$$

Equation of motion

Solving the equation of motion $-x = \frac{d^2x}{dt^2}$

We want to solve this equation starting with $x = 1.0$ and $v_x = 0.$. We are going to check our solution by measuring the position at intervals of ϵ.

Filling up the table

- Where will the particle be at a small time interval $\Delta t = \epsilon$ latter?
  - $$x(\epsilon) = x(0) + v_x(0)\,\epsilon = 1.0 + 0.0\epsilon = 1.0$$
- Where will the particle be at $t = 2\epsilon$?
  - $$x(2\epsilon) = x(\epsilon) + v_x(\epsilon)\,\epsilon = 1.0 + v_x(\epsilon)\,\epsilon$$
  - But we don't know $v(\epsilon)$. We are stuck.

t	x	v
0	1.0	0.0
ϵ	1.0	?
2ϵ	?	?
3ϵ	?	?

Table: Motion of the mass on a spring

Dynamical meaning of the equation: $-x = \frac{dv}{dt}$

Equation of motion allows us to calculate the new velocity

$$
\begin{aligned}
v(\epsilon) &= v(0) + a(0)\,\epsilon \\
&= v(0) + (-x(0))\epsilon
\end{aligned}
$$

t	x	v
0	1.00	0.000
0.1	1.00	-0.100
0.2	0.990	-0.200
0.3	0.970	-0.299

Table: Motion of the mass on a spring with $\epsilon = 0.1$