Lecture - 1: Motion

Workshop on Mechanics and Python

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References

This lecture is based on
Chapter 8 of The Feynman Lectures in Physics, Volume 1

Description of motion

- We would like to find the laws governing the changes that take place in a body as time goes on.
- We will start with some solid object like a cycle. We will mark the head of the cyclist and call it a point and follow the motion of the point in time.

Description of motion along a line

- Let us consider even more simpler situation in which our cycle moves along a straight line only.
- Then we can describe the motion by giving the position of our marker, the point, on the straight line as time flows.

Description of motion as a table

t (min)	s (ft)
0	0
1	120
2	400
3	900
4	950
5	960
6	1300
7	1800
8	2350
9	2400

Table: Position of the cycle at different instances of time

Description of motion as a graph

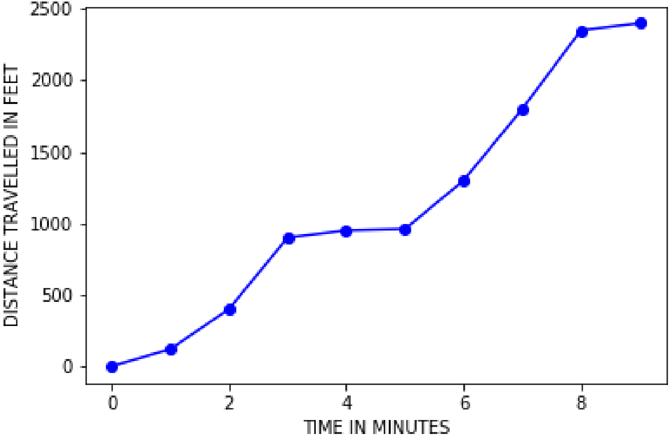


Figure: Motion of the cycle as a graph of distance versus time

Another view of the motion

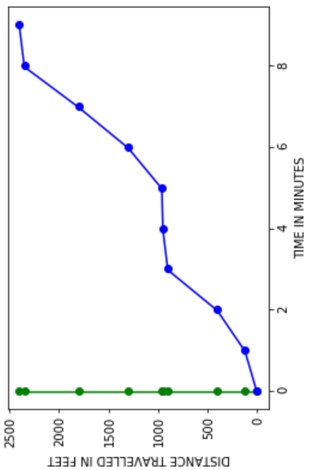


Figure: Motion on the line and its graph as a function of time

A formula describing a motion

- If we know the laws of nature that describe the motion of a particle in one dimension, then we should be able to give the position of the particle at every instant of time.
- We can write this as

$$x = f(t)$$

A falling object

- Using Newton's laws of motion, and knowing the initial condition of the particle, we can obtain a formula that describes the motion of a falling object

$$S = \frac{1}{2}gt^2; \quad g = 9.8\frac{m}{s^2} \tag{1}$$

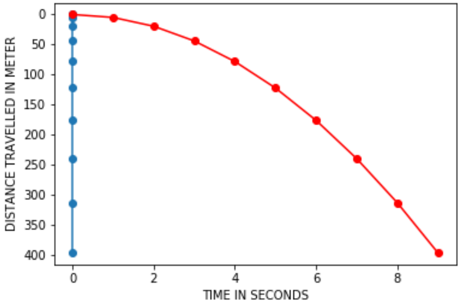


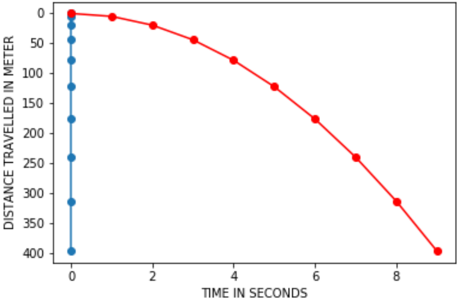
Figure: Falling body

Space and Time

- Our description of motion is based on some intuitive assumptions about space and time.
- These assumptions have to be reexamined when:
  - Particles are moving at a speed comparable to the speed of light.
  - When the size of object is "small".

How fast is the particle moving?

- We have an intuitive notion of a particle moving slow or fast.
- For falling boy we can "see" that it moves slowly first and then moves faster.



Speed

Let us define the speed at time t as:

$$V(t) = \frac{S(t + \Delta t) - S(t)}{\Delta t}$$

- What is Δt?
- Since speed is changing Δt should be "small".
- Will speed at time t depend on our choice of Δt?

Speed at $t = 0$ for a falling body: $S = \frac{1}{2}gt^2$

- We will measure our speed to the precision of five decimal places

Δt	$V(0) = \frac{S(0+\Delta t)-S(0)}{\Delta t}$
0.01	0.04900
0.001	0.00490
0.0001	0.00049
0.00001	0.00005
0.000001	0.00000
0.0000001	0.00000

$V(0)$

For $\Delta t < 1 \times 10^{-5}$ the speed $V(0)$ is independent of Δt till five decimal places.

Speed at $t = 0$ with increased precision

- Let us measure our speed to the precision of **six** decimal places

Δt	$V(0) = \frac{S(0+\Delta t)-S(0)}{\Delta t}$
0.001	0.004900
0.0001	0.000490
0.00001	0.000049
0.000001	0.000005
0.0000001	0.000000
0.0000001	0.000000

$V(0)$

For $\Delta t < 1 \times 10^{-6}$ the speed $V(0)$ is independent of Δt till six decimal places.

Speed from the formula

The results of our numerical experiments can be obtained for $S = \frac{1}{2}gt^2$

- $$V(t) = \frac{S(t + \Delta t) - S(t)}{\Delta t}$$
- $$V(t) = \frac{\frac{1}{2}g\left(\left(t^2 + 2t\Delta t + \Delta t^2\right) - t^2\right)}{\Delta t}$$
- $$V(t) = gt + \frac{1}{2}g\Delta t$$

Speed as a Derivative

$v\left(t\right)$ the speed at time t

$$v\left(t\right) = \lim_{\Delta t \to 0} \frac{S\left(t + \Delta t\right) - S\left(t\right)}{\Delta t} \equiv \frac{dS}{dt}$$

Table of Velocity

- We want to describe the motion assuming we know the speed of the particle at every time.
t	$v(t)$
0	0
1	9.8
2	19.6
3	29.4
4	39.2
- We want to know what is $S(4)$, where the particle will be at 4sec?

Finding the position from the velocity

- From our definition of velocity, with $\Delta t = 1\text{sec}$, and assuming $S(0) = 0$

$$
\begin{aligned}
S(1) &= S(0) + v(0)\Delta t = 0 \\
S(2) &= S(1) + v(1)\Delta t = 0 + 9.8 \\
S(3) &= S(2) + v(2)\Delta t = 9.8 + 19.6 = 29.4 \\
S(4) &= S(3) + v(3)\Delta t = 29.4 + 39.2 = 68.6
\end{aligned}
$$

- Check

$$S(4) = \frac{1}{2}9.8 \times 4^2 = 78.4 \neq 68.6!$$

- Velocity is changing and $\Delta t = 1$ is too large!

Distance as a sum

- $$S(4) = v(0)\,\Delta t + v(1)\,\Delta t + v(2)\,\Delta t + v(3)\,\Delta t$$
- $$S(4) = \sum_{i=1}^{4} v(i-1)\,\Delta t$$
- In general

$$S(T) = \sum_{i=1}^{N} v(i-1)\,\Delta t; \quad T = N\Delta t$$

Distance as a sum: $\Delta t \to 0$

- To take into account that the velocity is changing in the interval Δt we must make Δt smaller and smaller
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Δt	$S(4)$
1.0	68.6
0.001	78.380
0.0001	78.398
0.00001	78.399
0.000001	78.400

Distance as an integral

Distance is a definite integral of $v(t)$

$$S(T) = \lim_{\Delta t \to 0} \sum_{i=1}^{N} v(i-1)\,\Delta t; \quad T = N\Delta t$$

$$S(T) \equiv \int_0^T v(t)\,dt$$