How Can We Say Something Without Knowing Every Thing?

An Introduction to Renormalization Group

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Outline

Two problems two responses

High Temperature Superconductivity

We do not understand superconductivity in many materials, particularly in so called high temperature superconductors. But no condensed matter physicist is asking for a high energy accelerator which will probe more carefully electron-electron and electron-nucleon interactions.

Higgs Mass

We do not understand the origin of Higgs mass. Many particle physicist would like to build a new particle accelerator to replace LHC. They feel the answer can be found only by probing with greater resolution the interaction between Higgs and other elementary particles.

Aim of this talk is to understand why two different approaches

We will try and develop a formalism to substantiate our intuitive approach, and see where this intuition may fail us.

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Why should equations have correct dimensions?

A dimensionally correct equation

$$A\,\text{meter} = B\,\text{meter}$$

Such an equation is **independent of the units used**

$$A\,\text{fermi} = B\,\text{fermi}$$

A dimensionally incorrect equation

$$A\,\text{meter} = B\,\text{kg}$$

Such an equation depends on the units used

$$A\,\text{fermi} \neq B\,\text{kg}$$

Scaling and Resolving

We expect dimensionally correct equation to scale

$$A\,\mathrm{s}\times\mathrm{meter} = B\,\mathrm{s}\times\mathrm{meter}$$

the relationships holds as we scale our **measuring rod** or equivalently when we change the **resolving power** of our measuring instrument.

How does a length of a curve scales?

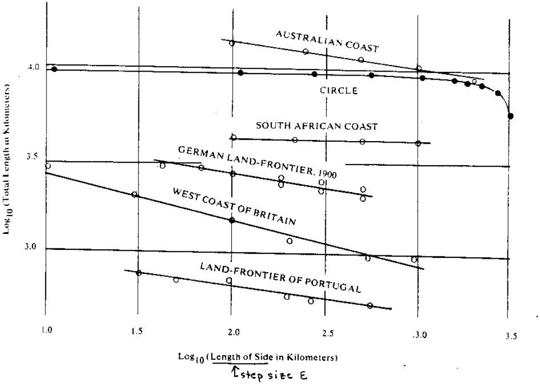


Plate 33 ¤ RICHARDSON'S EMPIRICAL DATA CONCERNING THE RATE OF INCREASE OF COASTLINES' LENGTHS

Scaling Dimension and Engineering Dimension

The engineering dimension of a quantity may not determine its scaling

$$l = N(\epsilon)\,\epsilon$$

ϵ is the step size. For coastlines Richardson's data suggests

$$l = \left(\frac{L}{\epsilon}\right)^{d} \epsilon = L^{d} \epsilon^{(d-1)}, \quad d \neq 1$$

$$\ln(l) = d \ln(L) - (d-1)\ln(\epsilon)$$

d which we all refer to as the scaling dimension is not equal to 1 which is the engineering dimension.

Dimensional Analysis in Natural Units: $c = 1$

$c = 1$

We will measure time in units of meter.

1 meter of time is the time it takes to travel one meter

$$c = 1 = \frac{1\text{m}}{\text{the time it takes light to travel one meter}} = \frac{1\text{m}}{1\text{m}} = 1$$

In these units c is dimensionless. We can go back to "human units" using

$$3\times 10^8 \text{m}\,\text{s}^{-1} = 1.$$

Now we have only two dimensional quantities

$$[\text{energy}]\,, \quad [\text{length}]\,.$$

Dimensional Analysis in Natural Units: $c = 1$ and $\hbar = 1$

$c = 1, \quad \hbar = 1$

$$\hbar = 197\text{GeV fm}$$

We choose to measure length in the units of inverse energy

$$1\text{fm} = \frac{1}{197\text{GeV}}.$$

In out new units value of $\hbar$ is 1 and it is dimensionless.

$$\hbar = 197\text{GeV fm} = 1\text{GeV}1\text{GeV}^{-1} = 1.$$

Natural Units

All physical quantities are measured in units of energy.

An Introduction to Quantum Mechanics

States and observables

Even when a system is in a definite state, all its physical properties are not well defined, but their average value are well defined and can be calculated knowing the state of the system

Average value of a physical quantity $P = < \Psi | \hat{P} | \Psi >$

Path integral formulation of quantum mechanics

All quantum averages can be calculated from the partition function Z of the system

$$Z = \sum_{\text{paths}} e^{iS[\vec{r}(t)]},$$

where $S\left[\vec{r}(t)\right]$ is the action of the system.

An Introduction to Quantum Field Theory-1

Quantum mechanics of systems with indefinite number of particles

- Many, if not the most, states of physical systems are made up of indefinite number of particles. Such systems cannot be described using "paths" of particles.
- We need a description in which a state has indefinite number of particles

$$|\Phi >= \sum_{n=0}^{\infty} c_n |n > .$$

An Introduction to Quantum Field Theory-2

Fields and Normal Modes

- A "free" field can be expanded in its normal modes.
- Each normal mode acts like a SHO.

Quantum Harmonic Oscillator

- Energy of quantum SHO is of the form

$$E_n = \left(n + \frac{1}{2}\right)\omega$$

- We can identify number of particles n in the state

$$|\Phi >= \sum_{n=0}^{\infty} c_n |n >$$

with the n^{th} excitation of a normal mode of a field

An Introduction to Quantum Field Theory-3

Quantum Field Theory

Quantum Field Theory is a quantum mechanics of fields $\Phi(x, y, z, t)$.

Path Integral Formulation of QFT

Like any quantum mechanical system the quantum averages can be calculated from the partition function

$$Z = \sum_{\{fields\}} e^{iS[\Phi]} = \int [\mathrm{d}\Phi]\, e^{iS[\Phi]}$$

An example of QFT

Action

Guess an action

$$S[\phi] = \int d^4x \left\{ \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 - V(\phi) \right\},$$

where $V(\phi)$ is a polynomial in ϕ

Calculate the partition function

Observe that

$$Z = \int [d\phi] \, e^{iS[\phi]} = \infty$$

but

$$Z[\Lambda] = \int [d\phi]_\Lambda \, e^{iS[\phi]} \neq \infty$$

QFT according to Dyson, Feynman, Schwinger, Tomonaga

Infinities

For a general $S[\phi]$

$$\lim_{\Lambda\to\infty} Z[\Lambda] = \infty$$

Renormalization

If

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \lambda \phi^4$$

then making m and λ dependent on Λ in a contrived manner that Feynman referred to as

"Sweeping infinities under the carpet"

a finite answers for quantum averages could be extracted in the limit $\Lambda \to \infty$

Dimension analysis in QFT

QFT in natural units

- All dimensional quantities will have dimension of energy or mass.
- $$[S[\phi]] = 0; \quad \text{Action is dimensionless}$$
- $$[x] = -1; \quad \text{Length has dimension of inverse mass}$$
- $$S[\phi] = \int d^4x \left\{ \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi) \right\},$$

  implies that

  $$[\phi] = 1; \quad [V(\phi)] = 4.$$

Physics is where the action is

What is $S[\phi]$?: Wilsonian Answer

- Quantum field theory is a system in which fields with arbitrary short wavelengths contribute to quantum averages.
- Let Λ_0 be the energy scale till which it makes sense to talk about space and time ($\Lambda_0 \sim M_{\text{plank}} = \sqrt{\frac{1}{G_{\textbf{Newton}}}}$)
- Write the most general form of the action that is consistent with the symmetries of the space-time

$$S_{\Lambda_0}[\phi; g_2, g_3, \dots] = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 - \left(\left(g_2 \Lambda_0^2 \right) \phi^2 + \left(g_3 \Lambda_0 \right) \phi^3 \right. \right.$$
$$\left. \left. + \left(g_4 \right) \phi^4 + \left(\frac{g_5}{\Lambda_0} \right) \phi^5 + \cdots \right) \right].$$

The action is defined by an infinite number of dimensionless parameters g_i and a cutoff Λ_0.

What about theory of some thing that we can probe?

Traditional approach

Ignore the short distance physics and write the simplest possible action

$$S_{\mathrm{naive}}[\phi, g_2, g_3, g_4; \Lambda_{\mathrm{Lab}}] = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 - \left(\left(g_2(\Lambda_{\mathrm{Lab}}) \Lambda_{\mathrm{Lab}}^2 \right) \phi^2 \right. \right.$$
$$\left. \left. + \left(g_3(\Lambda_{\mathrm{Lab}}) \Lambda_{\mathrm{Lab}} \right) \phi^3 + g_4(\Lambda_{\mathrm{Lab}}) \phi^4 \right) \right]$$

$$g_2(\Lambda_{\mathrm{Lab}}) = \left(\frac{\Lambda_0}{\Lambda_{\mathrm{Lab}}} \right)^2 g_2(\Lambda_0),$$
$$g_3(\Lambda_{\mathrm{Lab}}) = \left(\frac{\Lambda_0}{\Lambda_{\mathrm{Lab}}} \right) g_3(\Lambda_0),$$
$$g_4(\Lambda_{\mathrm{Lab}}) = g_4(\Lambda_0).$$

Don't ignore integrate it out

Separate the fluctuations that you can resolve from that you can't

$$
\begin{aligned}
\phi(x) &= \int_{|p|<\Lambda_0} \exp(ipx)\,\phi(p) \\
&= \int_{|p|<\Lambda_{\mathbf{Lab}}} \exp(ipx)\,\phi(p) + \int_{\Lambda_{\mathbf{Lab}}<|p|<\Lambda_0} \exp(ipx)\,\phi(p) \\
&= \bar{\phi}(x) + \tilde{\phi}(x)
\end{aligned}
$$

Integrate out the stuff you can't resolve

$$
\begin{aligned}
Z &= \int [d\bar{\phi}] \left(\int \left[d\tilde{\phi}\right] \exp\left(iS\left[\phi, g_i(\Lambda_0); \Lambda_0\right]\right) \right) \\
&= \int [d\bar{\phi}] \exp\left(iS_{\text{Wilson}}\left(\bar{\phi}, g_i(\Lambda_{\text{Lab}}); \Lambda_{\text{Lab}}\right)\right)
\end{aligned}
$$

So What?

Wilsonian Effective Action

$$S_W[\phi, g_i(\Lambda); \Lambda] = \int d^4x \left\{ \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - \sum_{n=2}^{\infty} g_n(\Lambda)\Lambda^{(4-n)}\phi^n \right\}$$

What was the effect of integrating out the high energy modes?

$$g_2(\Lambda) = g_2(\Lambda_0)\left(\frac{\Lambda_0}{\Lambda}\right)^2 + \int [d\tilde{\phi}] \exp\left(\cdots i g_4(\Lambda_0) \int \phi\phi\tilde{\phi}\tilde{\phi}\cdots\right)$$

In general we will have

$$g_n(\Lambda) = \left(\frac{\Lambda}{\Lambda_0}\right)^{n-4} + \text{"quantum correction due to high energy modes"}$$

Note that for $n > 4$ the coupling constant gets suppressed but for the quantum correction.

The Flow Equations

The unknown physics effects the probed physics through the flow equations

$$\Lambda \frac{dg_n}{d\Lambda} = (n-4)\, g_n + \beta_{\text{quantum}}\,(g_i)$$

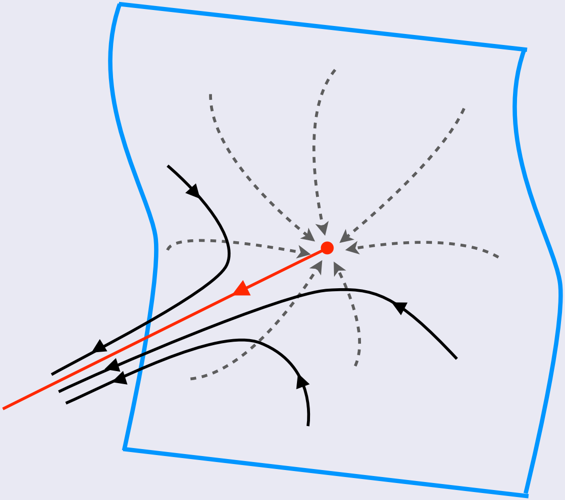
Is it relevant?

$$\Lambda \frac{dg}{d\Lambda} > 0 \quad \text{"Irrelevant at lab energies"}$$
$$\Lambda \frac{dg}{d\Lambda} = 0 \quad \text{"Important at all scales - determines the flow"}$$
$$\Lambda \frac{dg}{d\Lambda} < 0 \quad \text{"Relevant at lab energies"}$$

Physics is possible because there are only finite number of relevant couplings

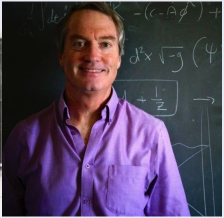
The Big Picture

Follow the flow

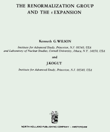


Wilsonian View: A cartoon of the history in three pictures

Some People



Some Response


THE RENORMALIZATION GROUP
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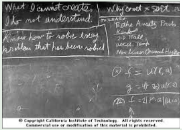

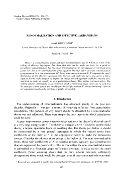
RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI

1. Introduction

QCD: Or how to get mass out of nothing

QCD action

$$S_{QCD}\left[A_\mu, \psi, g_{QCD}\left[\Lambda\right]; \Lambda\right]$$

Has only one (marginally) relevant coupling constant, g_{QCD}, which is dimensionless. **QCD action has no dimension-full parameter**.

What determines the mass of proton?

$$M_{\mathrm{proton}} = \Lambda f\left(g\left(\Lambda\right)\right)$$

There is a hidden dimensional parameter, the scale at which you define your coupling constant.

The problem of Naturalness

What determines the value of the Higgs mass?

- Values of relevant couplings "accumulates" as we flow from $M_{\text{planck}} \sim 10^{23}\text{GeV}$ to $M_{\text{LHC}} \sim 10^{4}\text{GeV}$
- Renormalization group flow of a relevant coupling, like mass

$$\mu_{\text{observed}} = \left(\frac{M_{\text{plank}}}{M_{\text{LHC}}}\right)^2 g_2\left(M_{\text{plank}}\right) f\left(g_i\right),$$

  where $f\left(g_i\right)$ is a dimensionless function of the original couplings.

- To get the observed value we have to tune the values of the coupling at the Planck scale with an extraordinary precision

$$\mu_{\text{observed}} = 125 = 10^{361} g_0 f\left(g_{io}\right),$$

  which seems unnatural!

When can Wilsonian View Fail?

Needs a fixed space-time background

- Our ability to separate between small and large requires a metric.
- Therefore it is not clear what happens if we want to describe gravity quantum mechanically.

Conclusion

Known unknowns

Renormalization group approach is a framework for organizing known unknowns. In nature there are, almost certainly, unknown unknowns!